Using Cardano's method for solving cubic equation [11], we get:

$$F = -\frac{\beta^\star}{2}\frac{(2P-1)^2}{3P^2+(P-1)^2}, \tag{30}$$

where for P we have:

$$P = \frac{1}{2}\sqrt[3]{\left(\frac{\beta^\star + \sqrt{\beta^{\star 2} - \alpha^{\star 2}}}{\alpha^\star}\right)^2}. \tag{31}$$

some

After computation, is for F we get:

$$F = -\frac{\beta^\star}{2}\frac{\left(\sqrt[3]{\alpha^{\star 2}} + \sqrt[3]{(\beta^\star + \sqrt{\beta^{\star 2} - \alpha^{\star 2}})^2}\right)^3}{\alpha^{\star 2} + (\beta^\star + \sqrt{\beta^{\star 2} - \alpha^{\star 2}})^2}. \tag{32}$$

substituting

~~Replacing~~ now $\beta^* = b^i p_i$ and $\alpha^{*2} = p_i p^i = a^{ij} p_i p_j$ we can easyly get (20).
If $b^2 \neq 1$ (28) is more complicated because:

$$F = \frac{\beta^* t^2}{-2t^2 + 3t + b^2 - 1}, \tag{33}$$

by substituting   obtain   ← obtain

and ~~puting~~ this in (25) we ~~get~~ the quadric equation:

$$t^4 - 3t^3 + t^2\frac{13-4b^2}{4} + t\frac{6\alpha^{*2}(b^2-1)}{4\alpha^{*2}} + \frac{\alpha^{*2}(b^2-1)^2 + \beta^{*2}(1-b^2)}{4\alpha^{*2}} = 0. \tag{34}$$

a quite long computation

After ~~laborious calculs~~, ~~[11]~~, (34) formula becomes a cubic equation (different from (29))and solving ~~this~~ it we get:

$$\begin{aligned} F &= -\frac{\beta^*}{2}\Bigg(\Bigg(\Bigg(\sqrt{-A^2+3A+2\sqrt{A^2+m\Big(b^2-\frac{\beta^{*2}}{\alpha^{*2}}\Big)}} + \frac{A}{2} + \frac{3}{4}\Bigg)^2 \\ &+ \sqrt{A^2+m\Big(b^2-\frac{\beta^{*2}}{\alpha^{*2}}\Big)} - \frac{5}{4}\Big(A+\frac{3}{10}\Big)^2 + n\Bigg) / \\ &/ \Bigg(\Big(\frac{3}{2}+2A\Big)\Big(\sqrt{-A^2+3A+2\sqrt{A^2+m\Big(b^2-\frac{\beta^{*2}}{\alpha^{*2}}\Big)}} \\ &+ 2\sqrt{A^2+m\Big(b^2-\frac{\beta^{*2}}{\alpha^{*2}}\Big)} + \frac{9}{2}A + p\Bigg), \end{aligned} \tag{35}$$

where

$$A^2 = \sqrt[3]{\Big(\frac{1}{2}\frac{\beta^{*2}}{\alpha^{*2}} + \varepsilon_1\Big)^2 + \varepsilon_3} + \sqrt[3]{-4\Big(\theta_4^3\frac{\beta^{*2}}{\alpha^{*2}} + \varepsilon_2\Big)} + \theta_5. \tag{36}$$