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1. If $b^2 = 1$, the L-dual of (M, F) is a Kropina space on T^*M with:

$$H(x,p) = \frac{1}{2}\left(\frac{\alpha^{*2}}{2\beta^*}\right)^2. \tag{17}$$

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2. If $b^2 \neq 1$, the L-dual of (M, F) is a Randers space on T^*M with:

$$H(x,p) = \frac{1}{2}\left(\alpha^* \pm \beta^*\right)^2, \tag{18}$$

with $\alpha^* = \sqrt{\tilde{a}^{ij}(x)p_ip_j}$ where

$$\tilde{a}^{ij} = \frac{1}{1-b^2}a^{ij} + \frac{1}{(1-b^2)^2}b^ib^j; \quad \tilde{b}^i = \frac{1}{1-b^2}b^i$$

(in (18) $'-'$ corresponds to $b^2 < 1$ and $'+'$ corresponds to $b^2 > 1$).

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Theorem 2.2' The L-dual of a Kropina space is a Randers space on T^*M with the Hamiltonian:

$$H(x,p) = \frac{1}{2}\left(\alpha^* \pm \beta^*\right)^2, \tag{19}$$

where

$$\tilde{a}^{ij} = \frac{b^2}{4}a^{ij}; \quad \tilde{b}^i = \frac{1}{2}b^i$$

(in (19) $'-'$ corresponds to $\beta < 0$ and $'+'$ corresponds to $\beta > 0$).

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We are going to compute the dual of a Matsumoto metric. We obtain

Theorem 2.3 Let (M, F) be a Matsumoto space and $b = (a_{ij}b^ib^j)^{\frac{1}{2}}$ the Riemannian lengh of b_i. Then

1. If $b^2 = 1$, the L-dual of (M, F) is the space having the fundamental function:

$$H(x,p) = \frac{1}{2}\left(-\frac{b^ip_i}{2}\frac{\left(\sqrt[3]{p_ip_ja^{ij}} + \sqrt[3]{\left(p_ib^i + \sqrt{p_ip_j\tilde{a}^{ij}}\right)^2}\right)^3}{p_ip_ja^{ij} + \left(p_ib^i + \sqrt{p_ip_j\tilde{a}^{ij}}\right)^2}\right)^2, \tag{20}$$

where

$$\tilde{a}^{ij} = b^ib^j - a^{ij}.$$

2. If $b^2 \neq 1$, the L-dual of (M, F) is the space on T^*M having the fundamental function:

$$H(x,p) = \frac{1}{2}\left(-\frac{b^ip_i}{200}\frac{25\left(2\sqrt{p_ip_jd_2^{ij}} + \sqrt{p_ip_jd_4^{ij}}\right)^2 + p_ip_jd_8^{ij}}{\sqrt{p_ip_jd_2^{ij}}\sqrt{p_ip_jd_4^{ij}} + p_ip_jd_9^{ij}}\right)^2, \tag{21}$$