

Conversely,

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Thus, for the Cartan space  $(M, F)$  we always can locally associate a Finsler space  $(M, F)$  which will be called the L-dual of a Cartan space  $(M, C|_{U^*})$ . ~~Vice-versa~~, we can associate, locally, a Cartan space to every Finsler space which will be called the L-dual of a Finsler space  $(M, F|_U)$ .

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## 2 The $(\alpha, \beta)$ Finsler - $(\alpha^*, \beta^*)$ Cartan L-duality

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**Theorem 2.1** ([3, 8]) Let  $(M, F)$  be a Randers space and  $b = (a_{ij}b^ib^j)^{\frac{1}{2}}$  the Riemannian length of  $b_i$ . Then:

1. If  $b^2 = 1$ , the L-dual of  $(M, F)$  is a Kropina space on  $T^*M$  with:

$$H(x, p) = \frac{1}{2} \left( \frac{a^{ij} p_i p_j}{2b^i p_i} \right)^2. \quad (14) \quad (2.1)$$

2. If  $b^2 \neq 1$ , the L-dual of  $(M, F)$  is a Randers space on  $T^*M$  with:

$$H(x, p) = \frac{1}{2} \left( \sqrt{\tilde{a}^{ij} p_i p_j} \pm \tilde{b}^i p_i \right)^2, \quad (15) \quad (2.2)$$

where

$$\tilde{a}^{ij} = \frac{1}{1-b^2} a^{ij} + \frac{1}{(1-b^2)^2} b^i b^j; \quad \tilde{b}^i = \frac{1}{1-b^2} b^i,$$

(in (15) '-' corresponds to  $b^2 < 1$  and '+' corresponds to  $b^2 > 1$ ).

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**Theorem 2.2** ([3, 8]) The L-dual of a Kropina space is a Randers space on  $T^*M$  with the Hamiltonian:

$$H(x, p) = \frac{1}{2} \left( \sqrt{\tilde{a}^{ij} p_i p_j} \pm \tilde{b}^i p_i \right)^2, \quad (16) \quad (2.3)$$

where

$$\tilde{a}^{ij} = \frac{b^2}{4} a^{ij}; \quad \tilde{b}^i = \frac{1}{2} b^i,$$

(in (16) '-' corresponds to  $\beta < 0$  and '+' corresponds to  $\beta > 0$ ).

In [3]  $\alpha^* = (a^{ij}(x)p_i p_j)^{\frac{1}{2}}$ ,  $\beta^* = b^i(x)p_i$ , where  $a^{ij}(x)$  are the reciprocal components of  $a_{ij}$  and  $b^i(x)$  are the components of the vector field on  $M$ ,  $b^i(x) = a^{ij}(x)b_j(x)$ . We can consider the metric functions  $K = \alpha^* + \beta^*$  (Randers metric on  $T^*M$ ) or  $K = \frac{\alpha^{*2}}{\beta^*}$  (Kropina metric on  $T^*M$ ) defined on a domain  $D^* \subset T^*M$ . So, we can easily rewrite the previous theorems:

**Theorem 2.1'** Let  $(M, F)$  be a Randers space and  $b = (a_{ij}b^ib^j)^{\frac{1}{2}}$  the Riemannian length of  $b_i$ . Then:

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easily

the notations  
 $\alpha^* = \dots$ ,  $\beta^* = \dots$   
are used, where ...

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Nu folosiți "we" decât după rezultatele corecte din lucrare. Ceea ce am făcut alții și  
numai cități aici se specifică impersonal cu "one",