$\boxed{\text{FINAL PART}}$

Let's take a second look at formula (21). If we introduce the following quadratic forms

$$\alpha_2^* := d_2^{ij}(x)\, p_i p_j \ , \qquad \alpha_4^* := d_4^{ij}(x)\, p_i p_j$$

$$\alpha_8^* := d_8^{ij}(x)\, p_i p_j \ , \qquad \alpha_9^* := d_9^{ij}(x)\, p_i p_j$$

defined on T^*M ~~with~~ by the corresponding matrices, then (21) becomes

$$H(x, p) = \frac{1}{2}\left[\frac{\beta^*}{200} \times \frac{25\left(2\alpha_2^* + \alpha_4^*\right)^2 + \left(\alpha_8^*\right)^2}{\alpha_2^* \cdot \alpha_4^* + \left(\alpha_9^*\right)^2}\right]^2 \qquad (38)$$

for $b^2 \neq 1$.

In other words, ~~in the case~~ the $\mathcal{L}$-duals of a Randers and Kropina metrics ~~it~~ are expressed only with the duals α^*, β^* of α, β, respectively. However, ~~for~~ the $\mathcal{L}$-dual of a Matsumoto metric is given by means of four distinct quadratic forms on T^*M. Remark that the coefficients of these quadratic forms are constructed only from the Riemannian metric matrix elements $a_{ij}(x)$ and the 1-form β's coefficients $b_i(x)$.

The other properties like curvature and the relation between geometrical properties of the $\mathcal{L}$-dual metric (38) and the initial Matsumoto metric will be studied elsewhere.